$n_3 = 0$ in T_4, so that (16) may be written as

$$\psi_2(x,\xi,y,\eta) = \frac{-1}{2\pi\epsilon_0(1+\epsilon_r)} \log\{(x-\xi)^2 + (y-\eta)^2\} + H(x,\xi,y,\eta) \tag{17}$$

where $H(x,\xi,y,\eta)$ is continuous. For computational purposes this separation of the singularity is important.

For $b \to \infty$ (16) reduces to the open microstrip Green's function (12) whereas for $\epsilon_r = 1$ the only nonzero terms in (16) occur with $n_1 = n_2 = 0$ and it corresponds to the *in vacuo* case of a microstrip between ground planes.

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Field Propagation in an Open-Beam Waveguide

A. CONSORTINI, L. RONCHI, AND R. TOGNAZZI

***Abstract*—A new method is described for the investigation of the open-beam waveguides, which may be applied even when the optical elements are not inserted in absorbing screens.**

The theory of an open-beam waveguide deals, in general, with the determination of the iterative beams associated with it [1]. When the finite size of the optical elements composing the beam waveguide is taken into account, the iterative beams turn out to be strictly related to the oscillation modes of a suitable defined two-mirror open resonator, equivalent to the beam waveguide [2], [3]. Such an open resonator is composed of mirrors which behave in reflection like the optical elements of the waveguide behave in transmission.

An alternative method of treating the open-beam waveguides is to study how the field originates from a given source, at a given distance from the first guiding element, propagating from cell to cell. To this end, we can choose a plane per cell, for example, at the input of each guiding element, and evaluate the field at the points of the plane π_{n+1} in terms of the field at the points of the plane π_n (Fig. 1). By referring to a two-dimensional problem, where the quantities of interest depend only on the longitudinal coordinate z and on one transverse coordinate x, we can write, in the scalar approximation,

$$v_{n+1}(x_{n+1}) = \int_{-\infty}^{\infty} v_n(x_n) K_{n,n+1}(x_n,x_{n+1})\, dx_n \tag{1}$$

where v_i denotes the field (impinging) at the points of the plane π_i, x_i,z_i, the coordinates of the points of π_i, and $K_{i,i+1}$, the Green's function describing the propagation from the plane π_i to the plane π_{i+1}, including the transmission through L_i.

Starting with the source distribution $v_0(x_0)$, (1) allows us to evaluate $v_1(x_1)$, then $v_2(x_2)$, and so on, in other words, to study the evolution of the field through the waveguide. In the general case, the evaluation of the fields $v_i(x_i)$ must be done numerically, using an electronic computer. This implies some practical difficulty when the guiding elements are not inserted in absorbing screens. If the guiding elements are "diaphragmed" (Fig. 2), (1) can be written in the form

$$v_{n+1}(x_{n+1}) = \int_{a_n}^{b_n} v_n(x_n) K_{n,n+1}(x_n,x_{n+1})\, dx_n \tag{2}$$

where $b_n - a_n$ represents the aperture of the nth pupil, and the evaluation of v_{n+1} requires an integration over a finite interval, which is easily done by the electronic computers. When the pupil apertures have infinite width (only partially occupied by the guiding elements, in practical cases), the integration is to be made over an infinite interval, which creates problems of accuracy of the results.

It occurred to us, however, that this difficulty can be overcome in the following way. Let us denote by ξ_n and ξ_n' the limits of the region of the plane π_n occupied by L_n. Equation (1) may be rewritten in the form

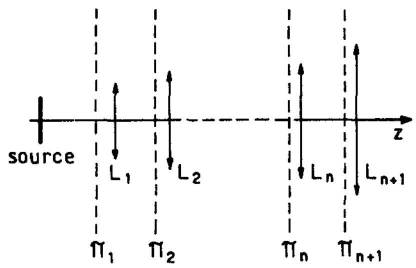

Fig. 1. A general beam waveguide composed of the elements L_i, with the reference planes π_i.

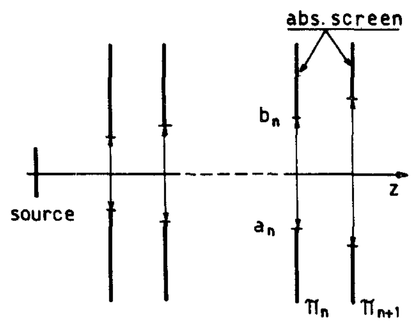

Fig. 2. A sequence of diaphragmed guiding elements.

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$$v_{n+1}(x_{n+1}) = \int_{-\infty}^{\xi_n} v_n(x_n) G_{n,n+1}(x_n,x_{n+1})\, dx_n + \int_{\xi_n}^{\xi_n'} v_n(x_n) G_{n,n+1}'(x_n,x_{n+1})\, dx_n + \int_{\xi_n'}^{\infty} v_n(x_n) G_{n,n+1}(x_n,x_{n+1})\, dx_n \quad (3)$$

where $G_{n,n+1}$ describes the free-path propagation from (x_n,z_n) to (x_{n+1},z_{n+1}), while $G_{n,n+1}'$ describes the transmission through L_n and the free-path propagation from L_n to π_{n+1}. Equation (3) can also be written as

$$\begin{aligned} v_{n+1}(x_{n+1}) &= \int_{-\infty}^{\infty} v_n(x_n) G_{n,n+1}(x_n,x_{n+1})\, dx_n + \int_{\xi_n}^{\xi_n'} v_n(x_n) [G_{n,n+1}'(x_n,x_{n+1}) - G_{n,n+1}(x_n,x_{n+1})] dx_n \\ &= \int_{-\infty}^{\infty} v_{n-1}(x_{n-1}) K_{n-1,n+1}'(x_{n-1},x_{n+1})\, dx_{n-1} + \int_{\xi_n}^{\xi_n'} v_n(x_n) [G_{n,n+1}'(x_n,x_{n+1}) - G_{n,n+1}(x_n,x_{n+1})] dx_n. \end{aligned} \quad (4)$$

Here $K_{n-1,n+1}'$ describes the propagation from π_{n-1} to π_{n+1} in the absence of L_n. In other words, the field impinging onto π_{n+1} can be derived from the field impinging onto the plane π_{n-1} and by means of an integration in the plane π_n limited to the region occupied by L_n. By iterating the procedure, we arrive at a formula for the field v_{n+1} by means of an integral in the plane π_0 of the source and of n integrals over the regions of the planes $\pi_1,\pi_2,\cdots,\pi_n$ occupied by $L_1,L_2,\cdots,L_n$. Thus for example,

$$v_1(x_1) = \int v_0(x_0) G_{0,1}(x_0,x_1)\, dx_0$$

$$v_2(x_2) = \int v_0(x_0) G_{0,2}(x_0,x_2)\, dx_0 + \int_{\xi_1}^{\xi_1'} v_1(x_1) [G_{1,2}'(x_1,x_2) - G_{1,2}(x_1,x_2)] dx_1$$

$$v_3(x_3) = \int v_0(x_0) G_{0,3}(x_0,x_3)\, dx_0 + \int_{\xi_1}^{\xi_1'} v_1(x_1) [G_{1,3}'(x_1,x_3) - G_{1,3}(x_1,x_3)] dx_1 + \int_{\xi_2}^{\xi_2'} v_2(x_2) [G_{2,3}'(x_2,x_3) - G_{2,3}(x_2,x_3)] dx_2$$

and so on. In this way, if the source is of limited size, as happens in all real cases, all the integrations involved in the evaluation of v_{n+1} are over finite intervals. If, on the contrary, the source is of infinite size, as may happen in some cases of theoretical interest, the free-path propagation from the source to any plane π_i is to be considered known (refer, for example, to the case when the field impinging onto the waveguide is a plane wave or a Gaussian beam).

As an example, we have applied the method to study the field propagation through a periodic sequence of dielectric frames [4], [5], illuminated either by a plane wavefront with the same size as the frames, or by a plane unlimited wave. Symmetry of the frames with respect to the z axis is also assumed.

Let us denote by $2a$ the inner width of the frames (Fig. 3), by l the width, by δ the optical thickness (given by $(\nu - 1)\delta'$, where ν denotes the refractive index of the dielectric and δ' the thickness), and by L the distance between two consecutive frames. In the computation, we have used the approximate expressions

$$G_{i,j}(x_i,x_j) = G(x_i,x_j)$$

$$G_{i,j}'(x_i,x_j) = \frac{\exp(-i\pi/4)}{(\lambda r)^{1/2}} \exp(ikr) \cos\theta$$

where $r = |P_iP_j|$, with $P_i \equiv (x_i,z_i)$, $P_j \equiv (x_j,z_j)$, and θ denotes the angle between $P_j - P_i$ and the z axis. The time dependence is assumed in the form $\exp(-i\omega t)$. Finally, we had to take into account the fact that the region of any plane π_i occupied by the guiding element is divided into two parts, from $-(a + l)$ to $-a$ and from a to $a + l$.

The results of the calculation are shown in Figs. 4 to 8, for $a = 22.36\ \lambda$, $l = 1.7\ \lambda$, $L = 100\ \lambda$, $\delta = 0.8\ \lambda$. Figs. 4 and 5 refer to the

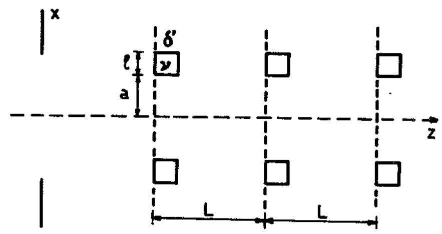

Fig. 3. A periodic sequence of dielectric frames.

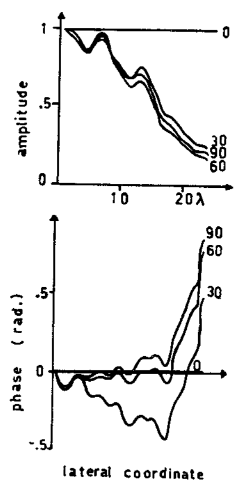

Fig. 4. Normalized field amplitude and phase at several frames, plotted versus x, when the source is a plane wavefront truncated at $|x_0| = a + l$, at a distance 100 λ from the first element.

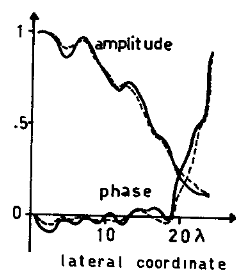

Fig. 5. The quasi-stationary field configuration (solid lines) and the iterative field of the open-resonator theory (dashed lines), for the same case as Fig. 4.

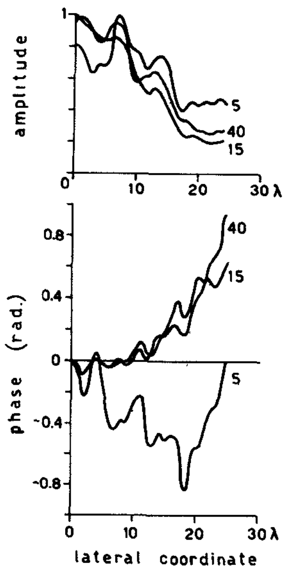

Fig. 6. Case of an impinging unlimited plane wave: normalized field amplitude and phase, plotted versus x, at several frames.

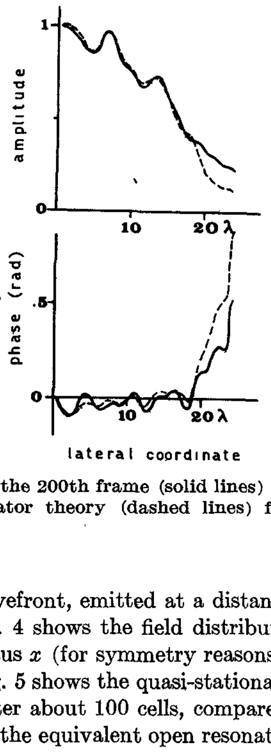

Fig. 7. The field at the 200th frame (solid lines) and the iterative field of the open-resonator theory (dashed lines) for the same case as Fig. 6.

plane truncated wavefront, emitted at a distance $L = 100\ \lambda$ before the first frame. Fig. 4 shows the field distribution at a number of frames, plotted versus x (for symmetry reasons, the field is plotted only for $x \geq 0$). Fig. 5 shows the quasi-stationary field distribution, which is reached after about 100 cells, compared with the iterative field distribution of the equivalent open resonator theory [6], [7].

Figs. 6 and 7 refer to an impinging plane wave. Dashed lines represent, as before, the iterative field.

Fig. 8 shows the quantities Φ_n/Φ_1 and Φ_n/Φ_{n-10}, plotted versus n, where

$$\Phi_n = \int_{-(a+l)}^{a+l} |v_n(x_n)|^2 dx_n$$

represents the power flux through the nth frame.

Our treatment of open-beam waveguides has a general character, so that it can be applied not only to periodic but also to nonperiodic sequences of optical elements. Thus it is suitable for the study of the effects of errors in the position (both in the longitudinal and in the transverse senses) of the elements, and of differences in the optical properties of them, as well as of possible bends in the axis of the beam waveguide (the planes π_i are not necessarily parallel to one another).

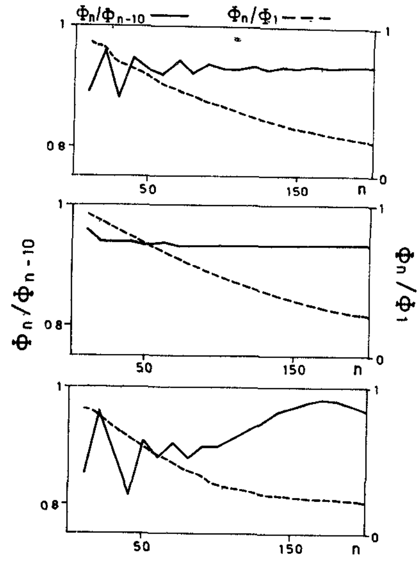

Fig. 8. The ratio Φ_n/Φ_{n-10} (solid lines) and the ratio Φ_n/Φ_1 (dashed lines), plotted versus n, when the beam waveguide is illuminated by a truncated plane wave (top), the iterative field of open resonator theory (center), and an unlimited plane wave (bottom).

In addition, it may be generalized so as to take into account the backscattering of the optical elements, which was neglected in (5).

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Application of the Beam Mode Expansion to the Analysis of Noise Reduction Structure

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***Abstract*—The beam mode expansion method used to discuss the diffraction problem by an aperture is applied to the analysis of the noise reduction structure consisting of two aperture stops. The incident field is a fundamental wave beam whose amplitude distribution is Gaussian. The transmitted field through the structure can be represented as a sum of beam mode functions and is regarded as a signal. The noise which is originated from the spontaneous**

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